NAG Fortran Library Routine Document

C06PKF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

C06PKF calculates the circular convolution or correlation of two complex vectors of period n.

2 Specification

```
SUBROUTINE C06PKF(JOB, X, Y, N, WORK, IFAIL)
INTEGER            JOB, N, IFAIL
complex            X(N), Y(N), WORK(2*N+15)
```

3 Description

This routine computes:

if JOB $= 1$, the discrete **convolution** of x and y, defined by

$$z_k = \sum_{j=0}^{n-1} x_j y_{k-j} = \sum_{j=0}^{n-1} x_{k-j} y_j;$$

if JOB $= 2$, the discrete **correlation** of x and y defined by

$$w_k = \sum_{j=0}^{n-1} \bar{x}_j y_{k+j}.$$

Here x and y are complex vectors, assumed to be periodic, with period n, i.e., $x_j = x_{j\pm n} = x_{j\pm 2n} = \ldots$; z and w are then also periodic with period n.

Note that this usage of the terms 'convolution' and 'correlation' is taken from Brigham (1974). The term 'convolution' is sometimes used to denote both.

If $\hat{x}$, $\hat{y}$, $\hat{z}$ and $\hat{w}$ are the discrete Fourier transforms of these sequences, and $\tilde{x}$ is the inverse discrete Fourier transform of the sequence x_j, i.e.,

$$\hat{x}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(-i\frac{2\pi jk}{n}\right), \text{ etc.,}$$

and

$$\tilde{x}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(i\frac{2\pi jk}{n}\right),$$

then $\hat{z}_k = \sqrt{n}.\hat{x}_k\hat{y}_k$ and $\hat{w}_k = \sqrt{n}.\bar{\hat{x}}_k\hat{y}_k$ (the bar denoting complex conjugate).

This routine calls the same auxiliary routines as C06PCF to compute discrete Fourier transforms.

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice-Hall

5 Parameters

1: JOB – INTEGER *Input*

On entry: the computation to be performed:

if JOB = 1, $z_k = \sum_{j=0}^{n-1} x_j y_{k-j}$ (convolution);

if JOB = 2, $w_k = \sum_{j=0}^{n-1} \hat{x}_j y_{k+j}$ (correlation).

Constraint: JOB = 1 or 2.

2: X(N) – ***complex*** array *Input/Output*

On entry: the elements of one period of the vector x. If X is declared with bounds $(0 : \mathrm{N} - 1)$ in the (sub)program from which C06PKF is called, then $\mathrm{X}(j)$ must contain x_j, for $j = 0, 1, \ldots, n - 1$.

On exit: the corresponding elements of the discrete convolution or correlation.

3: Y(N) – ***complex*** array *Input/Output*

On entry: the elements of one period of the vector y. If Y is declared with bounds $(0 : \mathrm{N} - 1)$ in the (sub)program from which C06PKF is called, then $\mathrm{Y}(j)$ must contain y_j, for $j = 0, 1, \ldots, n - 1$.

On exit: the discrete Fourier transform of the convolution or correlation returned in the array X.

4: N – INTEGER *Input*

On entry: n, the number of values in one period of the vectors X and Y. The total number of prime factors of N, counting repetitions, must not exceed 30.

Constraint: N $\geq$ 1.

5: WORK(2∗N+15) – ***complex*** array *Workspace*

6: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

IFAIL = 2

On entry, JOB $\neq$ 1 or 2.

IFAIL = 3

IFAIL = 4

7 Accuracy

The results should be accurate to within a small multiple of the ***machine precision***.

8 Further Comments

The time taken by the routine is approximately proportional to $n \times \log n$, but also depends on the factorization of n. The routine is somewhat faster than average if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

9 Example

This program reads in the elements of one period of two complex vectors x and y, and prints their discrete convolution and correlation (as computed by C06PKF). In realistic computations the number of data values would be much larger.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     C06PKF Example Program Text.
*     Mark 19 Release. NAG Copyright 1999.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=64)
*     .. Local Scalars ..
      INTEGER          IFAIL, J, N
*     .. Local Arrays ..
      complex          WORK(2*NMAX+15), XA(NMAX), XB(NMAX), YA(NMAX),
     +                 YB(NMAX)
*     .. External Subroutines ..
      EXTERNAL         C06PKF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'C06PKF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
   20 CONTINUE
      READ (NIN,*,END=80) N
      WRITE (NOUT,*)
      IF (N.GT.1 .AND. N.LE.NMAX) THEN
         DO 40 J = 1, N
            READ (NIN,*) XA(J), YA(J)
            XB(J) = XA(J)
            YB(J) = YA(J)
   40    CONTINUE
         IFAIL = 0
*
         CALL C06PKF(1,XA,YA,N,WORK,IFAIL)
         CALL C06PKF(2,XB,YB,N,WORK,IFAIL)
*
         WRITE (NOUT,*) '        Convolution            Correlation'
         WRITE (NOUT,*)
         DO 60 J = 1, N
            WRITE (NOUT,99999) J - 1, XA(J), XB(J)
   60    CONTINUE
         GO TO 20
      ELSE
         WRITE (NOUT,*) 'Invalid value of N'
```

```
      END IF
   80 CONTINUE
      STOP
*
99999 FORMAT (1X,I5,2(:1X,'(',F9.5,',',F9.5,')'))
      END
```

9.2 Program Data

```
C06PKF Example Program Data
    9
      (1.0e0,-0.5e0)      (0.5e0,-0.25e0)
      (1.0e0,-0.5e0)      (0.5e0,-0.25e0)
      (1.0e0,-0.5e0)      (0.5e0,-0.25e0)
      (1.0e0,-0.5e0)      (0.5e0,-0.25e0)
      (1.0e0,-0.5e0)      (0.0e0,-0.25e0)
      (0.0e0,-0.5e0)      (0.0e0,-0.25e0)
      (0.0e0,-0.5e0)      (0.0e0,-0.25e0)
      (0.0e0,-0.5e0)      (0.0e0,-0.25e0)
      (0.0e0,-0.5e0)      (0.0e0,-0.25e0)
```

9.3 Program Results

```
 C06PKF Example Program Results

         Convolution            Correlation

     0 ( -0.62500, -2.25000) (  3.12500, -0.25000)
     1 ( -0.12500, -2.25000) (  2.62500, -0.25000)
     2 (  0.37500, -2.25000) (  2.12500, -0.25000)
     3 (  0.87500, -2.25000) (  1.62500, -0.25000)
     4 (  0.87500, -2.25000) (  1.12500, -0.25000)
     5 (  0.37500, -2.25000) (  1.62500, -0.25000)
     6 ( -0.12500, -2.25000) (  2.12500, -0.25000)
     7 ( -0.62500, -2.25000) (  2.62500, -0.25000)
     8 ( -1.12500, -2.25000) (  3.12500, -0.25000)
```